



Finiteness conditions and cotorsion pairs

Marco A. Pérez

Instituto de Matemáticas de la UNAM

October 02, 2015

Joint work with Daniel Bravo (Universidad Austral de Chile)

Introduction and motivation

Let R be an associative ring with identity, and M be a left R -module.

(FP0) M is said to be **finitely generated** if there exists a finite set I and an epimorphism

$$R^{(I)} \rightarrow M \rightarrow 0$$

(FP1) M is said to be **finitely presented** there exists an exact sequence

$$\underbrace{F_1 \rightarrow F_0 \rightarrow M}_{1\text{-presentation}} \rightarrow 0$$

such that F_0 and F_1 are finitely generated and free.

Introduction and motivation

Let R be an associative ring with identity, and M be a left R -module.

(FP0) M is said to be **finitely generated** if there exists a finite set I and an epimorphism

$$R^{(I)} \rightarrow M \rightarrow 0$$

(FP2) M is said to be **finitely 2-presented** there exists an exact sequence

$$\underbrace{F_2 \rightarrow F_1 \rightarrow F_0 \rightarrow M \rightarrow 0}_{2\text{-presentation}}$$

such that F_0 , F_1 and F_2 are finitely generated and free.

Introduction and motivation

Let R be an associative ring with identity, and M be a left R -module.

(FP0) M is said to be **finitely generated** if there exists a finite set I and an epimorphism

$$R^{(I)} \rightarrow M \rightarrow 0$$

(FP n) M is said to be **finitely n -presented** there exists an exact sequence

$$\underbrace{F_n \rightarrow F_{n-1} \rightarrow \cdots \rightarrow F_2 \rightarrow F_1 \rightarrow F_0 \rightarrow M}_{n\text{-presentation}} \rightarrow 0$$

such that F_0, F_1, \dots, F_n are finitely generated and free.

Introduction and motivation

Let R be an associative ring with identity, and M be a left R -module.

(FP₀) M is said to be **finitely generated** if there exists a finite set I and an epimorphism

$$R^{(I)} \rightarrow M \rightarrow 0$$

(FP_∞) M is said to be **finitely ∞ -presented** there exists an exact sequence

$$\underbrace{\cdots \rightarrow F_2 \rightarrow F_1 \rightarrow F_0 \rightarrow M \rightarrow 0}_{\text{infinite presentation}}$$

such that F_k is finitely generated and free, for every $k \geq 0$.

$$\begin{array}{rcccl}
& & \dots \longrightarrow & Q_1 \longrightarrow \text{Ker}(f_0) \longrightarrow 0 \longrightarrow \dots & \in \widetilde{(\mathcal{P}_n)}^\perp \\
& & & \parallel & \downarrow & \downarrow \\
Q_\bullet & = & \dots \longrightarrow & Q_1 \longrightarrow Q_0 \longrightarrow 0 \longrightarrow \dots & \in \text{dg}\widetilde{\mathcal{P}_n} \\
\downarrow p & & & \downarrow & \downarrow f_0 & \downarrow \\
S^0(M) & = & \dots \longrightarrow & 0 \longrightarrow M \longrightarrow 0 \longrightarrow \dots & &
\end{array}$$

Thank you for your attention!

These slides, plus upcoming paper (containing details omitted here)
available from:

maperez.info

Now you may
have some coffee!

